

# 機構之構造合成

顏鴻森

國立成功大學機械工程學系暨研究所

台南市大學路一號

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## 摘要

本文針對過去三十年來有關機構構造合成之研究做一文獻回顧，包括設計方法與圖論在運動鏈與機構數目合成之運用。為方便說明起見，文中並從文獻中舉圖以參考。

( 關鍵詞：機構；構造合成；創造性設計；圖論 )

## Structural Synthesis of Mechanisms

Hong-Sen Yen

Dept. of Mechanical Engineering, National Cheng Kung University

1, Da-Hsueh Rd., Tainan, Taiwan

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### ABSTRACT

This paper reviews research articles on the structural synthesis of mechanisms, including design methodologies and applications of graph theory for the enumeration of kinematic chains and mechanisms, in the past thirty years. Several diagrams from various literature are shown for references.

(KEY WORDS: mechanism; structural synthesis; creative design; graph theory)

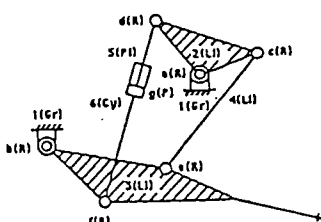
### INTRODUCTION

The most important stage in the design of mechanisms is the conceptual phase. Its

goal is to synthesize all possible topological structures of mechanisms and no dimensions are implied at this stage, This is the process of both type and number syntheses.

We call it structural synthesis of mechanisms or creative design of mechanisms. The topological structure of a mechanism, refers to the types, numbers, adjacency, and incidence of links and joints in the mechanism, and can be represented by its corresponding mechanism topology matrix(MTM)<sup>[13]</sup>. The MTM of a mechanism with N links is an N X N matrix. Diagonal element  $a_{ii}$  represents the type of member i. If

member i is adjacent to member k, non-digagonal element  $a_{ik}$  on the upper right represents the type of joint incident to members i and k, and that on the lower left represents the name of the joint. If member i is not adjacent to member k, then  $a_{ik}=0$ . For the mechanism shown in Figure 1(a), its corresponding MTM is shown in Figure 1(b).



$$MTM = \begin{pmatrix} Gr & R & R & 0 & 0 & 0 \\ a & Li & 0 & R & R & 0 \\ b & 0 & Li & R & 0 & R \\ 0 & c & c & Li & 0 & 0 \\ 0 & d & 0 & 0 & Pi & P \\ 0 & 0 & f & 0 & g & Cy \end{pmatrix}$$

(a)

(b)

Fig. 1. Mechanism topology matrix(MTM)<sup>[13]</sup>

The purpose here is to survey the developments of structural synthesis of mechanisms after 1960. This includes the design methodologies and how graph theory is involved.

## DESIGN METHODOLOGY

The efforts for creating the topological structures of mechanisms have been a long history in kinematics research. In 1964, Dobrjanskij and Freudenstein<sup>[4]</sup> summarized design steps for this goal based on numerous research works done before 1960. In the past three decades, two major systematic approaches regarding this subject were developed. One is based on the ideas of generalization, and the other is according to

the concept of separation of structure from function.

In 1965, Johnson and Towfigh<sup>[18]</sup> presented the concepts of equivalent kinematic chains and associate linkages for obtaining different mechanisms. Hall<sup>[14]</sup> explored this approach and proposed he ideas of generalization and respecialization. Yan and Hsu<sup>[27]</sup>, in 1983, introduced design requirements and constraints into this methodology. In 1988, Yan and Hwang<sup>[28]</sup> provided a complete set of generalizing rules for the generalization of mechanisms. In 1991, they derived algorithms for the precise specialization of mechanisms based on permutation groups and Polya theory<sup>[29]</sup>. In 1992, Yan<sup>[30]</sup> summarized this design approach and provided a methodology for the systematic generation of all possible

topological structures of mechanisms subject to requirements and constraints. This methodology has applications<sup>[2,19-20,31-33]</sup> in planetary gear trains for various transmissions and in suspension mechanisms. Fig 2 shows the application of this design methodology for synthesizing motorcross rear suspension mechanisms<sup>[30]</sup>.

The methodology based on the separation of structure from function was first proposed by Buchsbaum and Freudenstein<sup>[1]</sup>

in 1970 for the classification and enumeration of geared kinematic chains and other mechanisms. This concept was further presented by Freudenstein and Maki<sup>[8]</sup> in 1979, and has been widely used in different cases<sup>[7,9-10,5]</sup> such as planetary gear trains, window regulating mechanisms, engine mechanisms, and robot grippers. Figure 3 shows the application of this approach for synthesizing variable stroke internal combustion engine mechanisms<sup>[9]</sup>.

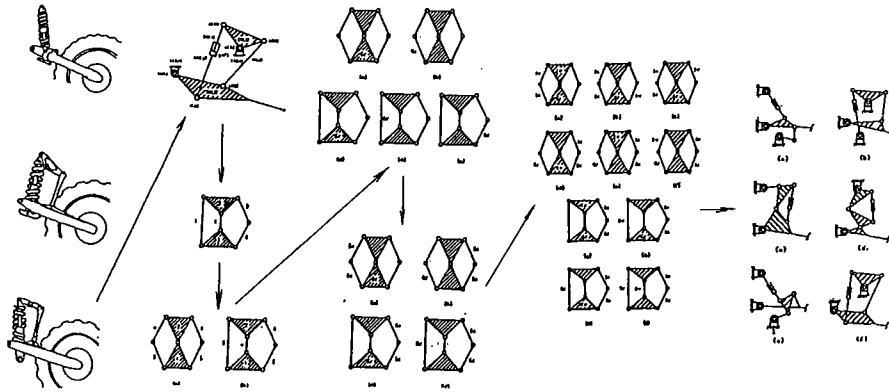


Fig. 2. Design methodology based on generalization and specialization<sup>[30]</sup>

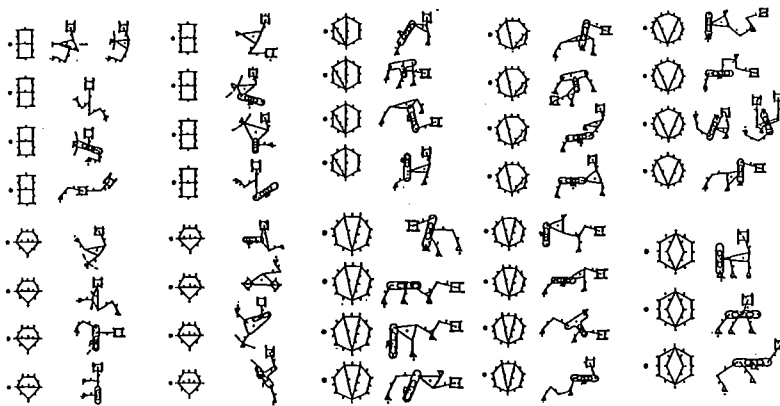


Fig. 3. Design methodology based on the separation of structure from structure<sup>[9]</sup>

## GRAPH THEORY

In the process of structural synthesis of mechanisms, it always requires the atlas of kinematic chains. This is number synthesis and has been the subject of numerous studies in the history of kinematics. One of the powerful tools for solving this problem is with the aid of graph theory. In what follows only major articles regarding the

application of graph theory for the number synthesis and structural synthesis of kinematic chains and mechanisms are reviewed.

Crossley<sup>[3]</sup> seems to be the first, in 1964, to have investigated the transformation between kinematic chains and graphs. He described a kinematic chain in terms of its dual interchange graph, and proved that there are precisely 16 kinematic chains with eight links and ten joints, Figure 4. In the same year, independently, Freudenstein and

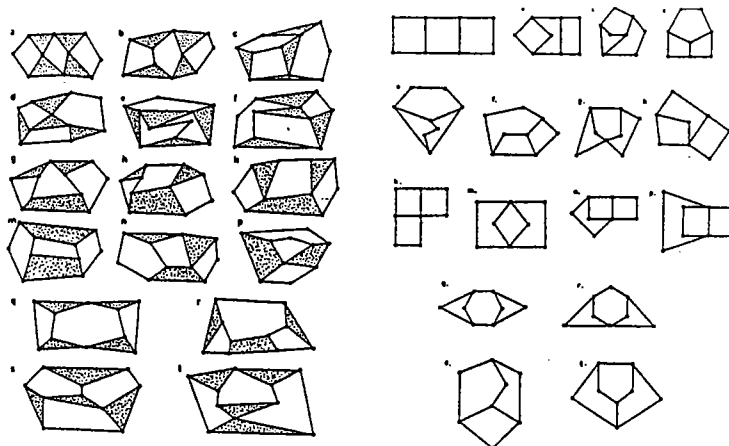


Fig. 4. Atlas of eight-link kinematic chains and its corresponding interchange graphs<sup>[3]</sup>

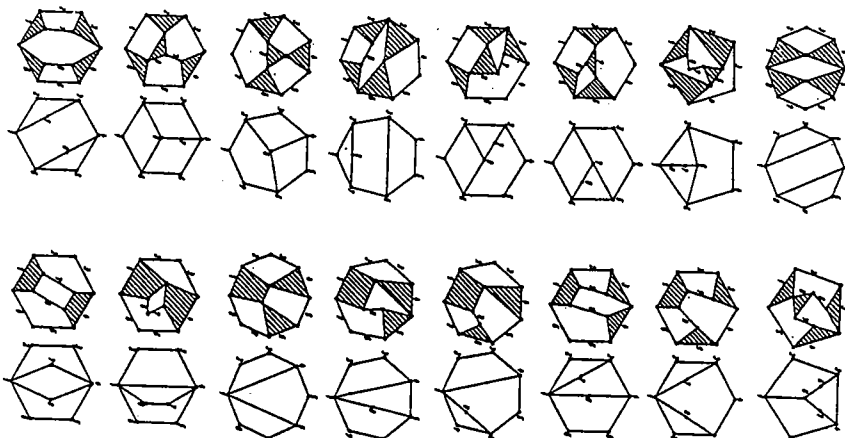


Fig. 5. Atlas of eight-link kinematic chains and its corresponding interchange graphs<sup>[4]</sup>

Dobrzanskyj<sup>[4]</sup> defined the graph of a kinematic chain for the enumeration of some basic mechanisms, Figure 5. They also used this concept to generate single-loop constrained spatial mechanisms<sup>[6]</sup> and geared kinematic chains<sup>[1]</sup>. In 1967, Freudenstein<sup>[11]</sup> reviewed preliminary concepts of Polya's theory and graphs for the structural classification of mechanisms. In the same year, Woo<sup>[26]</sup> applied the concept of the contraction map of a graph and synthesized 230 kinematic chains with ten links and thirteen joints. Huang and Soni<sup>[15-16]</sup>, in 1973, used the ideas of linear graphs, nonlinear graphs, and colored graphs with trees for the generation of kinematic chains with different elements. In 1974, Freudenstein and Woo<sup>[12]</sup> discussed the application of graph theory in the kinematic structure of mechanisms. Furthermore, Freudenstein and Maki<sup>[8]</sup> listed 57 closed graphs and Mayourian and Freudenstein<sup>[21]</sup> provided 35 admissible graphs with three to six points for kinematic chains. In 1986, Sohn and Freudenstein<sup>[24]</sup> used dual graphs to enumerate kinematic chains with up to eleven links. In 1987 and 1988 Oslon, Erdman, and Riley<sup>[22-23]</sup> addressed a new graph theory representation for the topological analysis of planetary gear trains. In 1988, based on the atlas of contracted graphs by Tempea joints

N <sup>F</sup>	1	2	3	4	5	6	7
6	2						
7		4					
8	16		7				
9		40		10			
10	230		98		14		
11		839		189		19	
12	6862		2442		354		24

Fig. 6. Number of kinematic chains with 6 to 12 links<sup>[35]</sup>

up to ten links. Furthermore, in 1990, Yan and Hwang<sup>[35]</sup> synthesized the number and atlas of nonisomorphic simple kinematic chains based on permutation groups, Figure 6.

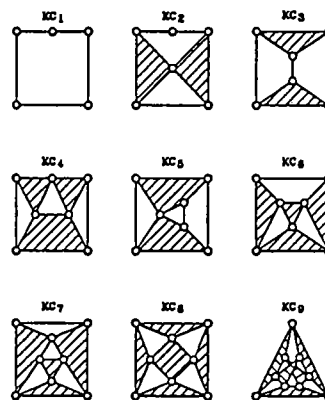


Fig. 7. Atlas of kinematic chains with five links where KCi has the planar block Bi as its graph<sup>[17]</sup>

The corresponding graph of a kinematic chain can best be represented by the planar block in graph theory. The term kinematic chain refers to an assemblage of interconnected links that is connected, closed, and without any cut link. In 1990, Harary and Yan<sup>[17]</sup> formulated a precise structural characterization of kinematic chains with simple and multiple joints based on the concepts from the theory of graphs and hypergraphs. They proved that each planar block produces a unique kinematic chain, and every kinematic chain has a unique associated graph that is a planar block, Figure 7.

## SUMMARY

The creation of design configurations is the most exciting and challenging task in

the process of designing mechanisms. The two design methodologies reviewed in this paper are quite valuable for industrial applications. Furthermore, the structural formulation of kinematic chains based on the ideas of graphs, line graphs and hypergraphs has tremendous possibilities to lead to new applicable concepts, and results for both kine-

matic chains and graph theory. In conclusion, the mentioned methodologies are power tools for design engineers not only for the systematic generation of all possible topological structures of mechanisms, but also for avoiding existing designs that have patent protections.

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